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Thermal Characterization of Honeycomb Core Sandwich Structures

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Honeycomb core sandwich structures are an integral part of many of today's aerospace structures. The need to know the thermal properties of these structures to perform thermal stress analyses provides the motivation for this research. The estimation approach used here to determine the thermal properties involves the minimization of an objective function containing both measured and calculated temperature values. A one-dimensional conductive/radiative heat transfer model was used for the analysis of the structure. Experimental designs for the collection of temperature response data were optimized using a scaled confidence interval approach. The experimental parameters optimized were the heating time and the total experiment time. Experiments were conducted at temperatures ranging from 295 to 495 K. The thermal properties estimated at these temperatures included the facesheet capacity, the conduction area of the core, and the emissivity in the interior of the core. These parameters exhibited a great deal of correlation or interaction. A penalty function method was used to perform the parameter estimation in a constrained fashion, and it was found that the parameters could be simultaneously estimated despite the presence of correlation.

Nomenclature

A_c	= conduction area of the core, m^2
$b, \boldsymbol{b}$	= estimated parameters (vector)
C_f	= volumetric heat capacity of the facesheet, J/m^{3}K
$\boldsymbol{H}$	= approximate second-order matrix for search direction
N	= total number of measurements
$P(\boldsymbol{b})$	= penalty function
$\boldsymbol{P}_{kk}$	= vector containing the diagonal terms of $[\boldsymbol{X}^T\boldsymbol{X}]^{-1}$
p	= number of parameters estimated
r_p	= penalty scaling parameter
S	= sum of squares function
$\boldsymbol{s}$	= search direction vector
$T, \boldsymbol{T}$	= calculated temperature (vector), K
$t_{1-\alpha/2}(N-p)$	= Student's t distribution
$\boldsymbol{X}$	= sensitivity coefficient matrix
$\boldsymbol{Y}$	= vector of measured temperatures, K
ϵ	= emissivity
Φ	= objective function

Introduction

THE purpose of this study was to develop a method for the estimation of thermal properties in honeycomb core sandwich structures. The estimation procedure that was used involved the minimization of an objective function containing both calculated and measured temperatures. Thus, both a mathematical model and an experimental setup were needed. The material being examined was a four sheet, superplastically formed/double bonded sandwich panel, composed of a Ti-6Al-4V alloy interior with aluminum-boron composite facesheets. To achieve the goals of this research, the specific objectives were 1) to develop a conductive/radiative finite element model for heat transfer through the thickness of the structure, 2) to optimize the experimental design and to conduct experiments to measure temperature using this design, 3) to formulate a parameter estimation algorithm to determine the desired properties, and 4) to estimate the unknown thermal properties in the heat transfer model.

The first objective deals with finding an appropriate heat transfer model for the structure. Because conduction and radiation heat transfer both exist within the structure, a combined model was developed. A one-dimensional finite element routine was used for the solution due to complexities caused by nonlinear radiation terms. Infrared imaging was used to verify that a one-dimensional model was sufficient to model the heat transfer within the structure.

The second objective is related to the experiments used to provide data for the parameter estimation procedure. The goal of most experiments is to produce results that have the highest possible accuracy and lowest variability. Some experimental design optimization is usually necessary to accomplish this. Here, experimental parameters were optimized to maximize the sensitivity of measured temperature data to changes in the unknown thermal properties to yield the smallest confidence intervals for these unknown properties.

Finally, in developing the estimation procedure for the thermal properties, an objective function was formulated that contained calculated data from the model and measured data from an actual experiment in a least-squares sense with a penalty function added to impose constraints on the estimated properties. A procedure was then implemented to minimize the objective function for the estimation of the unknown thermal properties in the sandwich structure.

Literature Review

Much work has been done in the area of parameter estimation and optimization. Many methods have been developed that are often specific to a certain field, with none being a standard. Thermal property estimation has been performed extensively on isotropic and composite materials. However, there are few cited cases of thermal property estimation dealing with honeycomb sandwich structures. What little work that has been done in this area has been mostly analytical in nature. Note that because the interest here lies in structures, rather than individual components, only methods allowing for simultaneous estimation of parameters are of interest. These methods generally require experiments to provide temperature data for the estimation procedure. Thus, another important aspect is the design of experiments to provide adequate information for the estimation procedure.

A carefully designed experiment is one in which there is minimum correlation between the estimated properties, as well as maximum sensitivity of the measured experimental variables to changes in the properties being estimated.[1] The design process involves the

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determination of optimal experimental variables through the use of a design criterion. The D-optimal criterion for experimental design has been widely used and studied. It was used by Beck[2] to optimize experimental parameters for the simultaneous estimation of thermal conductivity and specific heat. Taktak et al.[3] used the procedure to optimize heating time, sensor location, and number of sensors for the estimation of volumetric heat capacity and thermal conductivity in carbon-fiber/epoxy-matrix composite materials. Bayard et al.[4] used the D-optimal criterion to optimize sensor location for identification experiments in large space structures, while Moncman et al.[5] used this criterion to derive experimental designs for thermal property estimation in anisotropic composites.

Once the experiments have been designed and the data collected, a parameter estimation procedure is still needed. Parameter estimation can basically be treated as an optimization problem in which an objective function is minimized. The objective function in this case is typically a sum of squares function (S) in which measured and calculated dependent variables, e.g., temperature, are compared in a known model. Although parameter estimation has typically been performed with only a few different minimization methods, conceivably any optimization technique could be used.

Two estimation procedures were used in this research. The first method considered was the Gauss method. This is a first-order unconstrained descent method that minimizes a least-squares function containing differences between measured and calculated temperatures. It has the drawback of being inefficient for use in models that have correlated parameters or a nearly flat sum of squares function, as it may not converge. Box and Kanemasu[6] proposed modifications to this method to change the step size of gradient information used in finding the estimates at each iteration to improve convergence. This method was further modified by requiring that S be strictly decreasing.[7] The Gauss method and its derivatives are described in detail by Beck and Arnold.[1] The second procedure investigated was a penalty function method. This is a constrained optimization technique in which a penalty is added to the objective function if the estimated parameter exceeds the set constraints. Details are provided by Vanderplaats.[8]

The first method has been used by several researchers for the estimation of thermal properties. Beck[9] used the Gauss method to simultaneously estimate the thermal properties of nickel from transient temperature measurements. It was used successfully by Loh and Beck[10] to estimate thermal conductivities in two directions, along with volumetric heat capacity. This study also included information from multiple sensors within the sample. Six thermal properties were estimated by Pfahl and Mitchell[11] in a charring carbon-phenolic material. The related Box–Kanemasu method was used by Moncman,[12] Hanak,[13] and Scott and Beck[14] for the estimation of thermal properties in carbon/epoxy composites. This technique has also been used in the estimation of transport properties associated with biomedical applications[15] and the freezing of basic food solutions.[16]

No cases could be found where penalty function methods or other constrained techniques were used to solve parameter estimation problems. However, these methods have been used to solve finite element models and other forward problems. Ezawa and Okamoto[17] used penalty functions in conjunction with the finite element and boundary element methods to study elastic contact stresses. A similar approach was used by Fuehne and Engblom[18] to predict stress fields in thick composite laminates. Penalty functions were also used to solve the Navier–Stokes equations for heat transfer and fluid flow through an interface.[19]

When looking at the specific application of using parameter estimation techniques for the determination of thermal properties associated with a honeycomb core structure, very little information could be found in the literature. The mechanical and thermal characteristics have been investigated on an analytical basis; however, a majority of these analyses were mechanical.

One of the earliest thermal analytical works on honeycomb core sandwich panels was done by Swann and Pittman.[20] They used a one-dimensional finite difference heat transfer model that considered conductive and radiative effects. The model was steady state in nature and did not treat the thermal capacitance of the structure. The core was analyzed on a unit cell basis, with each cell divided into 10 elements. Constant temperature conditions were used at each boundary. The heat flux conducted through the structure was then calculated using known conductive and radiative properties of the structure material. An effective thermal conductivity was determined by Fourier's law. A similar analysis was performed for corrugated core panels.

Heat Transfer Model

A mathematical model of the heat transfer through the honeycomb core structure was needed to determine temperature histories necessary for the estimation of the thermal properties. Note that in the model one-dimensional heat transfer was assumed through the thickness of the structure. The model was developed using a general thermal-mechanical finite element code.[21]

The honeycomb structure consists of two distinct parts, the facesheet and the core, as shown in Fig. 1. A lumped capacitance model was used for the facesheet, with the volumetric specific heat C_f, i.e., the product of specific heat and density, to be determined. This assumption was possible for two reasons. First, the facesheet thickness was negligible compared with the height of the core. Second, the aluminum-boron composite that composed part of the facesheet had a thermal conductivity that was an order of magnitude higher than the rest of the structure. This permitted quick diffusion of heat throughout the facesheet.

The core was modeled as a flat plate with resistance and capacitance and a cross-sectional area equal to that of one cell. Because the specific heat and thermal conductivity of the core material, Ti-6Al-4V, are well known, attempting to estimate these properties would only serve to unnecessarily complicate the estimation procedure. The most important feature of the core is the wall thickness, which varies due to the fabrication procedure; therefore, its associated conduction area A_c was the property of interest. The core walls also include a series of holes and welds that were assumed to be an insignificant part of the heat transfer and were, therefore, not modeled separately. Thus, only an average wall thickness was estimated.

Air is also present in the core, and an analytical study was performed on the structure to determine its significance. Although based on this analysis the effect was expected to be small, heat conduction through the air was included in the model. However, free convection was neglected because its effect was expected to be minimal.

It was also desired to estimate the radiative characteristics of structure. A preliminary steady-state analysis indicated a significant radiative contribution to the overall heat transfer, and a final analysis verified this was true. (See the Results and Discussion section for final results.) In modeling the radiation component, it was assumed that the interior of the core represented a diffuse gray body, and as a result, only the total hemispherical emissivity ϵ was estimated.

A problem arose in how to model radiation heat transfer in a three-dimensional enclosure while the conduction was modeled as one dimensional. Radiation elements in the core were chosen to be equal area strips that ran horizontally from the bottom facesheet to the top, as shown in Fig. 1. The height of each element corresponded

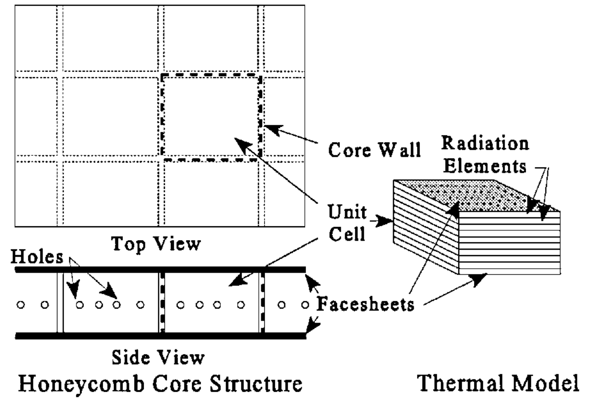

Fig. 1 Honeycomb core structure and mathematical model used in heat transfer analysis.

to a conduction element. The parts of the facesheets that enclosed the core were also treated as radiation elements.

To use radiation in the finite element code, it was necessary to use the view factor method. The view factor F_{ij} is defined as the fraction of radiant energy leaving surface i that arrives at surface j. Several texts list formulas for simple three-dimensional view factors. These, along with view factor algebra, could have been used to determine the view factors. However, this is a time-consuming process and also makes changing the number of elements impractical.

A code was written that uses a Monte Carlo method to calculate the view factors. Ten million rays were emitted from each of the core elements with a proportionally larger number emitted from the larger area of the facesheet. The coordinate axes were aligned with the facesheet and the sides of the core. A random location of emission on the facesheet was chosen by generating three random numbers, one for each coordinate. For the vertical elements, two random numbers, the height and the distance from the vertical corner, determined the location of emission. Two more random numbers were generated to determine a circumferential and azimuthal angle of emission.

Because of the geometry of the core, it was necessary to determine the intersection of each emitted ray with each plane of the core. The actual point of intersection was the one that was located within the dimensions of the enclosure. The specific element could then be determined by its distance from the bottom facesheet. A counter was incremented each time an element was hit by an emitted ray. The fraction of rays intercepted by surface j and emitted by surface i is the view factor F_{ij} as previously defined. View factors were calculated by emitting rays from the bottom facesheet element and half of the core elements. It was only necessary to emit rays from half of the elements because the enclosure is symmetric from top to bottom. The remaining view factors were assigned the same values as their symmetric counterpoints.

To verify the solution, the view factor from one facesheet to another was checked against the exact solution found for aligned parallel rectangles, Table 13.2 in Ref. 22. The exact solution from Ref. 22 gave a value of 0.3134. The Monte Carlo solution yielded 0.3138, a difference of only 0.13%. Energy was also found to be conserved overall. These facts lend credibility to the calculated view factors.

In the model, two boundary conditions were needed. A heat flux was provided at one of the boundaries to simultaneously estimate A_c and C_f. This boundary condition has the effect of introducing Fourier's law into the model, which provides information on A_c independent of C_f. For the second boundary, both a constant temperature and an insulated condition were initially investigated. However, it was later found that the insulated condition was very difficult to maintain experimentally, and so only the constant temperature boundary condition is considered here.

Thus, based on this model, the properties estimated in this study were the thermal capacitance of the facesheet C_f, the conduction area through the wall of the core A_c, and the emissivity in the interior of the core ϵ.

Sensitivity Analysis

Before conducting the experiments and estimating the properties, a sensitivity analysis was performed to gain insight into the overall estimation procedure. It is important that the experimental measurements provide adequate information or sensitivity for the estimation of the desired properties; that is, ideally this sensitivity should be high. Also, in the simultaneous estimation of multiple properties, it is important that the properties sought are not highly correlated, particularly if gradient-based methods are used. A sensitivity analysis can be used to assess both of these factors.

The sensitivity of the experimental measurements, which is temperature in this case, can be determined by calculating the dimensionless sensitivity coefficients, defined as $X_i^+ = b_{o,i}\partial T^+/\partial b_i$, where T^+ is dimensionless temperature, b_i ($i = 1$–3) are C_f, A_c, and ϵ, respectively, and the subscript 0 indicates a nominal value for each respective property. In general, one would like for $X_i^+ \geq 0.1$ for there to be sufficient information in the parameter estimation procedure and for the sensitivity coefficients to be linearly independent

Table 1 Maximum dimensionless sensitivity coefficients after heating for 1500 s

Property	Max. $\lvert X_i^+\rvert$	Time at max. $\lvert X_i^+\rvert$, s	Similarities in the shape of the curves
C_f	1.7	300	No similarities
A_c	2.2	1500	$X_{A_c}^+$ is similar to X_ϵ^+
ϵ	0.5	1500	X_ϵ^+ is similar to $X_{A_c}^+$

for the properties to be uncorrelated. The latter can easily be seen by plotting the sensitivity coefficients and examining their shapes to see if they differ significantly. Near linear dependence, which can be seen by similar but not identical curves, can also cause problems in the estimation procedure.

The dimensionless sensitivity coefficients for each unknown property were calculated as functions of time using a finite difference approximation. Note that a sensor located next to the heated surface provides the highest sensitivity in all cases. A summary of the results for the maximum X_i^+ values at the heated surface after 1500 s of heating is shown in Table 1. Several observations can be made from this analysis. First, the maximum sensitivity for ϵ (X_ϵ^+) was smaller than that for C_f and A_c ($X_{C_f}^+$ and $X_{A_c}^+$). Second, it appeared that ϵ and A_c were nearly correlated. These two observations suggested that ϵ in particular would be the most difficult property to estimate. Finally, the maximum sensitivity values for A_c and ϵ were at 1500 s, suggesting that these values are at a maximum at steady-state conditions (as expected), whereas the maximum value for C_f occurred at approximately 300 s, and then the magnitude of the sensitivity decreased toward zero as time increased. This behavior was also expected because C_f is associated with transient behavior.

Experimental Methods

Experimental Setup

The basic experimental setup, shown in Fig. 2, consisted of a thin resistance heater sandwiched between two honeycomb sample structures. The heater supplied the required heat flux boundary condition necessary for the simultaneous estimation of C_f and A_c. Large aluminum blocks were used at the two exposed boundaries to maintain the constant temperature boundary conditions. Temperature sensors were placed between the heater and each of the samples and between the aluminum blocks and the samples. Note that this configuration involved measurements that were noninvasive, and it was symmetric with respect to the heater. Thus, the heat flux supplied to the sample on either side of the heater was found from half of the total power input.

Each test sample measured $9.53 \times 12.7 \times 2.51$ cm thick. Each unit cell in the honeycomb sample measured $3.21 \times 4.31 \times 2.51$ cm thick. The core wall thickness was measured at various points and was approximately 0.046 cm. (Note that due to the fabrication procedure, the thickness is variable, and as a result, the conduction area was one of the properties estimated.) The aluminum blocks used for the constant temperature boundary condition measured $9.53 \times 12.7 \times 7.62$ cm thick. Aluminum was chosen because of its relatively high thermal conductivity and its availability. Insulation (360 Standard Ceramic Board, 5.08 cm thick) was placed around the edges of the setup to prevent heat loss in the lateral direction.

The temperatures measured at the heated surface provided the information necessary for the parameter estimation procedure. It was also necessary to record the temperature at the outer surfaces because the aluminum blocks did not act as perfect heat sinks. The resulting small temperature rise was recorded and incorporated into the estimation program as a variable temperature boundary condition.

There are several devices that can be used to measure temperature. Thermocouples are a popular choice. They were not, however, chosen for this study for several reasons. First, there would be grounding problems due to the metallic nature of the honeycomb structure. Second, thermocouples measure temperatures at a point. Because a one-dimensional model was used, point measurements were not necessary. Also, it would be difficult to locate thermocouples accurately on the facesheet with respect to the core components of the structure.

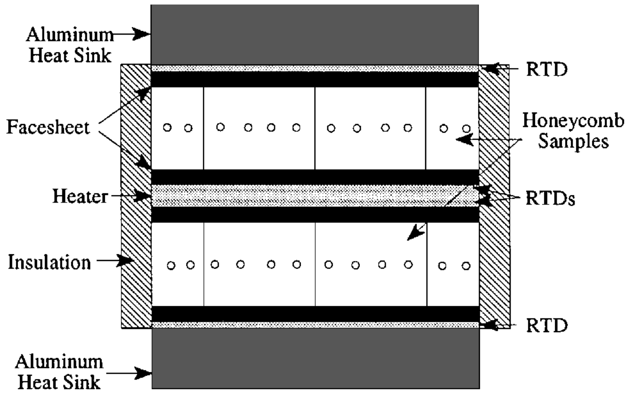

Fig. 2 Experimental setup for Ti-6Al-4V honeycomb sandwich structure.

The temperature sensor that was chosen was the resistance temperature detector (RTD), which is a resistance thermometer that responds positively to temperature. Four RTDs, the same cross-sectional area as the facesheet of the samples, were placed at each interface between the samples and the heater and the aluminum blocks as previously described. Each RTD contains platinum elements that have nominal resistance of 2000 Ω at 295 K (70°F).

Instrumentation was necessary to determine the resistance of the RTD at any temperature. The constant-current bridge was chosen as the instrumentation circuit because its output is more linear than the constant-voltage bridge. The three inactive arms of each bridge were completed with 2000 Ω ±0.005% resistors. It could then be assumed that the bridges were all balanced at 295 K, eliminating the need for a balancing potentiometer and allowing for the direct calculation of absolute temperature from a resistance change. The four bridges were placed in series and powered by a 1.0-mA current source.

Once the instrumentation for the RTDs was complete, the experimental apparatus was built starting with the heater, which had the same length and width as the samples. A thin layer of thermal joint compound was placed at each interface between the RTDs and the heater, the samples, and the aluminum blocks. The purpose of this was to reduce contact resistance.

The electrical resistance heater used in this experiment consisted of an electrical resistance element encased in Kapton®. The heater was rated at 10 Ω. The lead wires were externally attached to the heater to prevent added contact resistance at the heater surface. The voltage and current drawn by the heater were continuously measured. This was done to obtain an accurate reading of power consumption that might change over time due to variations in input voltage and resistance change as a result of heating. The heater was powered by a dc power supply capable of producing over 6 W. This was sufficient to make the heater output about 350 W/m^2.

The personal-computer-based data acquisition system used in this experiment consisted of both the data acquisition software and hardware. The purpose of the hardware was to bring voltage signals into the data acquisition program. There were six signals being recorded: the four RTD measurements, the voltage to the heater, and the current drawn by the heater. The RTD voltages were read directly from the bridge circuit by the hardware unit. The dynamic range of the system was always set to provide the highest possible resolution. Heater voltage was read from a digital multimeter. Current was read from a similar multimeter with the output voltage proportional to the current (1 V = 1 A).

A 7500-W oven was used in this experiment to provide an environment such that the thermal properties could be estimated as a function of temperature. The maximum temperature that was attained in any experiment was 495 K. No higher temperatures were used to avoid the possibility of destroying the Kapton coating of the heater and RTDs.

Optimization of the Experimental Design

It was desired to design the experiment such that the maximum amount of information could be obtained from the data for the estimation of the desired properties. The design parameters under consideration in this experiment were the heating time and the total experimental time. (Other factors, such as sensor location, were determined from previous experience.) The use of a finite heating time while taking additional data was warranted as it introduces a second transient in the system that is useful in estimating C_f. Thus, there is a balance to be determined between the properties most responsive at steady state (A_c and ϵ) and most responsive during transient phenomenon (C_f). The heating time and total experimental time were chosen to maximize the information for all three properties.

Many criteria have been proposed for the design of experiments, and these generally include some form of the $\boldsymbol{X}^T\boldsymbol{X}$ matrix, where $\boldsymbol{X}$ contains the dimensional sensitivity coefficients, that is, $X_i = \partial T/\partial b_i$, where b_i is the ith property estimated. The most common criterion is the maximization of the determinant of $\boldsymbol{X}^T\boldsymbol{X}$ (D-optimal). It will minimize the hypervolume of the confidence region but may do so at the expense of creating a large error in one of the parameters. In the end, it is desirable to make all the confidence intervals and regions as small as possible. A criterion, called the scaled confidence interval approach, was defined that seeks to minimize the largest confidence interval. The confidence interval is defined as

$$\Delta \boldsymbol{b}_k = \pm\{\boldsymbol{P}_{kk}[S/(N-p)]\}^{\frac{1}{2}} t_{1-\boldsymbol{\alpha}/2}(N-p) \tag{1}$$

where $\boldsymbol{P}_{kk}$ contains the time-averaged diagonal terms of the inverse of the $\boldsymbol{X}^T\boldsymbol{X}$ matrix and $t_{1-\boldsymbol{\alpha}/2}(N-p)$ is Student's t distribution for a $(1-\alpha/2)\times 100\%$ confidence interval. In using this criterion, $S/(N-p)$ is approximated by the square of the expected measurement error. Note that, with this approximation, all of the terms in Eq. (1) are constant except $\boldsymbol{P}_{kk}$; therefore, this criterion results in the minimization of $\boldsymbol{P}_{kk}$.

The optimization procedure consisted of first assuming values for the unknown properties in the mathematical model and then using the model to calculate finite difference approximations for the sensitivity coefficients to determine $\boldsymbol{P}_{kk}$ in Eq. (1). Then a simple parametric study was conducted in which the confidence intervals in Eq. (1) were calculated for each unknown property for different heating times and experimental times. The combination of heating and experimental times resulting in the lowest confidence intervals was chosen as the optimal design. The following values were used to determine the sensitivity coefficients: $\epsilon = 0.7$, $A_c = 3.5\times10^{-5}$ m^2, and $C_f = 27$ MJ/m^{3}K. (The value for A_c was based on actual measurements.) The scaled confidence interval approach yielded optimal heating times ranging from 3000 s at 295 K to 1500 s at 495 K and total experimental times ranging from 4600 s at 295 K to 2500 s at 495 K.

Experimental Procedures

Before running the experiments, thermal imagery was used to verify that the heat transfer was one dimensional and that the holes and welds in the core walls had a negligible effect, as assumed in the model. The samples were prepared in a manner similar to the way in which the experiments were run. The only exception was that one side of each sample was left exposed. The exposed side was painted black to have the highest possible emissivity. The sample was then heated to steady state and viewed with a thermal camera. The camera gave an infrared image of the sample. The camera was aimed at an angle that permitted a view of both a facesheet and the core. The major thermal gradient was in the direction normal to the heat flux, and the facesheet appeared to be nearly isothermal. Also, no significant effect was found at the holes. These observations served to validate the related assumptions used in the heat transfer model.

Experiments were then conducted from an initial temperature 295 K to 495 K in 50-K increments. Three replications were performed for each initial temperature. An experiment consisted of heating the sample for the optimized heating time, turning off the heat, and then continuing to collect data for the total optimized experimental time. Once again, this procedure allowed for additional information to be obtained for the estimation of C_f. All experiments were performed at the NASA Langley Research Center in Hampton, Virginia.

Parameter Estimation

The Gauss/Box–Kanemasu method was used first to estimate the unknown properties. However, the procedure did not converge. It was felt that this was due to the correlation between A_c and ϵ observed in the sensitivity study. Thus, another method was sought for the estimation procedure.

The finite element code used for the mathematical model also contained several optimization packages. It was convenient to use one of these because it allowed for easy interaction with the mathematical model. The routine chosen was a penalty function method that allowed for constraints to be imposed on the properties being estimated to keep the solution space away from areas of negative or physically impossible parameters. A brief overview of the method is provided here. The first step in the procedure was to establish an objective function. The objective function that was minimized was a modification of the standard sum of squares function S and is given by

$$\Phi(\boldsymbol{b}) = S + r_p P(\boldsymbol{b}) \tag{2}$$

where $S = (\boldsymbol{Y} - \boldsymbol{T})^T(\boldsymbol{Y} - \boldsymbol{T})$. The penalty function $P(\boldsymbol{b})$ is the square of inequality constraints. Thus, the constraints were incorporated directly into the objective function. The inequality constraints used here were that C_f and $A_c > 0$ and $0.4 < \epsilon \leq 1$. (This was based on the range of expected values.) The desired property estimates for C_f, A_c, and ϵ were those that minimized the objective function.

The next step, therefore, was to minimize the objective function in Eq. (2). This was achieved by setting the gradient of $\Phi(b)$ equal to zero. There are several ways of doing this; first-order methods are the most efficient. In the optimization package used, the Broyden–Fletcher–Goldfarb–Shanno method [8] was used to accomplish this. This is called a variable metric method in which prior information is contained in a p-dimensional array that is updated at each iteration and then a one-dimensional search is performed in each direction to find the minimum of the objective function. The search direction for each property s at each iteration k is determined by

$$\boldsymbol{s}^{(k)} = -\boldsymbol{H}^{(k)} \nabla \Phi\left[\boldsymbol{b}^{(k)}\right] \tag{3}$$

This equation results from taking the second-order Taylor series expansion of the objective function about $\boldsymbol{b}$, differentiating with respect to $\boldsymbol{b}$, and then setting the resulting expression equal to zero. Note that the coefficients of $\nabla\Phi[\boldsymbol{b}^{(k)}]$ contain the sensitivity coefficients, The second-order information is contained in $\boldsymbol{H}$, which is an approximation of the inverse of the Hessian matrix. Here $\boldsymbol{H}$ is derived from first-order approximations because a direct (second-order) calculation would be computationally expensive. In this method, $\boldsymbol{H}$ is initially set equal to the identity matrix (note that the initial search direction is that of steepest descent) and then updated at each iteration. Details of the procedures used to update $\boldsymbol{H}$ can be found in Ref. 8. Once the search directions for each property were determined, one-dimensional searches were performed to find the minimum of the objective function. The minimum was evaluated using polynomial interpolation.

Note that a variety of minimization methods could have been used. Both the Gauss-based method and the penalty function method can be thought of as gradient-based methods in which first the objective function is established and then minimized by setting the gradient of the objective function equal to zero. Thus, sensitivity coefficients were needed that were calculated using the finite element mathematical model as described previously. These were calculated at the sensor location and corresponding measurement times. Because of the nonlinearity of the problem, the minimization procedure was an iterative process in which initial estimates of the properties were first needed and then updated until a convergence criterion was met. Note that, because the sensitivity coefficients needed to be calculated using the mathematical model, it was necessary that the mathematical model could be used as a subroutine in the minimization procedure. Another approach would have been to use a nongradient-based method such as genetic algorithms (e.g., Refs. 23 and 24) in which the minimum of the objective function is found through direct evaluation and comparison. Note that these methods do not require calculation of the sensitivity coefficients.

Results and Discussion

Estimation of C_f, A_c, and ϵ

The results for the estimation of C_f, A_c, and ϵ are shown in Table 2. These results are also shown graphically in Fig. 3.

From the results shown in Table 2 and in Fig. 3, the confidence intervals for the emissivity estimates are much larger than those for the other three properties, i.e., most of these for ϵ are on the order of $\pm 20\%$ of the mean estimate. This is a result of the small sensitivity coefficients found for ϵ. The values, however, are consistent as all of the estimates fall within the confidence intervals of each other. The estimates at the two higher temperatures are seen to slightly decrease, suggesting a possible small temperature dependence of the emissivity at higher temperatures. Note also that a number of estimates at 345 and 395 K reached the upper constraint of $\epsilon = 1$ and that these have the largest confidence intervals.

The results for the estimation of A_c were expected to be independent of temperature. From Table 2, with the exception of the experiments at 345 K, the estimates once again fell within the confidence intervals of each other and were fairly consistent (within 20% for the worst case). Note also that the confidence intervals increased slightly as temperature increased. Recall that the conduction area was measured to be approximately 3.5×10^{-5} m^2. (Note that this is also an estimated value due to the variations in the wall thicknesses and the limited number of measurements taken.) From Table 2, the absolute percent deviations from this measured value and the mean of the estimated values at 295, 345, 395, 445, and 495 K are 6.6, 49, 7.7, 3.4, and 12.8%, respectively. Once again, with the exception

Table 2 Estimates of ϵ, A_c, and C_f at five different temperatures

Temp., K	Exp. no.	ϵ	$A_c \times 10^5$, m^2	$C_f \times 10^{-6}$, J/m^{3}K
295	1	0.93 ± 0.12	3.44 ± 0.25	2.87 ± 0.03
	2	0.85 ± 0.10	3.16 ± 0.21	3.00 ± 0.02
	3	0.86 ± 0.11	3.22 ± 0.22	3.00 ± 0.02
	Mean	0.88 ± 0.11	3.27 ± 0.23	2.96 ± 0.02
345	1	0.75 ± 0.25	5.31 ± 0.71	3.14 ± 0.07
	2	1.00 ± 0.28	5.14 ± 0.82	2.90 ± 0.08
	3	1.00 ± 0.28	5.20 ± 0.82	2.82 ± 0.08
	Mean	0.92 ± 0.27	5.22 ± 0.78	2.95 ± 0.08
395	1	1.00 ± 0.25	3.72 ± 0.99	3.18 ± 0.10
	2	1.00 ± 0.26	3.81 ± 1.01	3.15 ± 0.10
	3	0.96 ± 0.23	3.78 ± 0.94	3.16 ± 0.10
	Mean	0.99 ± 0.25	3.77 ± 0.98	3.16 ± 0.10
445	1	0.92 ± 0.25	3.85 ± 1.34	2.62 ± 0.14
	2	0.70 ± 0.20	3.45 ± 1.04	3.33 ± 0.11
	3	0.74 ± 0.22	3.56 ± 1.14	3.27 ± 0.12
	Mean	0.78 ± 0.23	3.62 ± 1.17	3.07 ± 0.12
495	1	0.62 ± 0.19	2.89 ± 1.21	3.07 ± 0.13
	2	0.65 ± 0.21	3.12 ± 1.37	3.06 ± 0.14
	3	0.65 ± 0.21	3.13 ± 1.37	3.09 ± 0.14
	Mean	0.64 ± 0.20	3.05 ± 1.32	3.07 ± 0.14

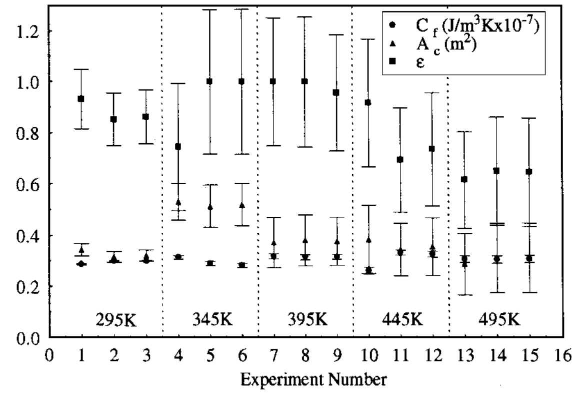

Fig. 3 Estimates of C_f, A_c, and ϵ with 95% confidence intervals at five different temperatures.

of the experiments run at 345 K, the estimates are very consistent with the measured value, lending credence to the estimation procedure.

The volumetric heat capacity resulted in estimates with the smallest confidence intervals. The mean estimates at each of the different temperatures varied from each other by less than 7%, indicating not only that the estimates were consistent but also that little evidence exists for any temperature dependence of C_f.

Analysis of the Mathematical Model

Once the property estimates were obtained, an analysis of the mathematical model using the estimated properties was conducted to determine the individual contributions by conduction and radiation to the overall heat transfer within the honeycomb core structure. It was found that, at steady state, radiation contributed over 25% of the total heat transfer at 295 K and approximately 40% at 495 K with the remaining percentages being attributed to conduction. Thus, radiation provided a significant portion of the overall heat transfer, supporting the inclusion of the detailed radiation analysis in the mathematical model.

General Observations

Some general comments can be made about the overall estimation procedure. First of all, this overall methodology provides a powerful tool in analyzing the thermal properties of honeycomb structures in general. To implement the procedure, one needs a mathematical model of the structure to calculate temperatures, experiments to provide measured temperatures, and an optimization procedure to estimate the desired properties. Second, this procedure is not dependent on the particular structure studied here, as a different mathematical model could easily have been used. Note that the accuracy obtained in using this procedure is partially dependent on the accuracy of the mathematical model used to simulate the experimental data. (As an aside note with regard to practicality, the mathematical model should be selected so that it can be interfaced with the optimization algorithm.) Also, optimization of the experimental design can be used to maximize the information obtained from the experiments for the estimation of the desired properties.

A sensitivity study is an important step in providing insight to the estimation problem. For example, it was found here that the sensitivity for emissivity was rather low, although radiation was found to contribute significantly to the overall heat transfer. The low-sensitivity coefficients resulted in large confidence intervals for ϵ. One can look at this result in two ways. First, this indicates that accurate estimates of this property are indeed difficult to obtain. On the other hand, the low sensitivity also suggests that highly accurate estimates are not needed to adequately model one-dimensional heat transfer through the structure for the temperature ranges studied. This is a very important point that is often overlooked. If another structure is to be studied with different materials and/or geometry, the sensitivity study can be very useful in determining which properties are most important to the heat transfer process and, therefore, need to be known with the greatest accuracy and in determining which properties have lesser influence and, consequently, can be used with lower levels of accuracy.

Summary and Conclusions

A methodology was developed and implemented for the estimation of unknown thermal properties of a honeycomb core sandwich structure. The overall methodology first involved the development of a one-dimensional mathematical model to determine the heat transfer, including conduction and radiation, through the structure. An experimental setup was constructed, and two experimental parameters were optimized. Experiments were then conducted at five different temperatures. The mathematical model and the experimental data were incorporated in a penalty function estimation procedure for the determination of the volumetric heat capacity of the facesheet, the emissivity in the interior of the core, and the conduction area in the walls of the core.

Overall, the resulting estimates were consistent from experiment to experiment. The confidence intervals associated with the emissivity were the largest, whereas those associated with the volumetric heat capacity were the smallest. None of the estimates at different temperatures indicated an overwhelming temperature dependence for the property in question. With the exception of the experiments at 345 K, the estimates for the conduction area were within 3–13% of measured values. This provided confidence in the estimation procedure.

Finally, the general procedure described here can be adapted to estimate the thermal properties of other types of honeycomb structures by incorporating the appropriate mathematical model for that structure into the procedure. A different optimization method can also be chosen; for example, if in the sensitivity study no correlation had been found between the properties being estimated, the Gauss method might have been used instead of the penalty function method. Thus, an important aspect of the estimation procedure in general is its overall versatility.

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